NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

AN INTEGRATED APPROACH TO THE DESIGN OF AN AIRCRAFT GAIN SCHEDULED CONTROLLER

by

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March, 1995

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
March, 1995

3. REPORT TYPE AND DATES COVERED
Engineer's Thesis

4. TITLE AND SUBTITLE
AN INTEGRATED APPROACH TO THE DESIGN OF AN AIRCRAFT GAIN SCHEDULED CONTROLLER

5. FUNDING NUMBERS

6. AUTHOR(S)
Erik Berglund

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Postgraduate School
Monterey, CA 93943

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES
The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the United States Government.

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

This thesis addresses the problem of integrated design of the aircraft plant parameters and of the corresponding feedback controller. The plant parameters are typically the sizes of the control surfaces or other aerodynamical surfaces of the aircraft. The approach is to rewrite the aircraft dynamic requirements as linear matrix inequalities (LMI's) and to optimize a linear cost function associated with aircraft plant parameters, while meeting the LMI constraints. An algorithm using Matlab and LMI-Lab has been developed. This algorithm has been used for integrated plant/controller design of an F-4 aircraft at five different flight conditions. The result consists of a set of controllers, one for each flight condition, and a single solution for the optimal sizes of the F-4 stabilator and spoiler control surfaces.

14. SUBJECT TERMS
Plant Optimization, Control Theory, Linear Matrix Inequalities

15. NUMBER OF PAGES
63

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std Z39-18
298-102

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AN INTEGRATED APPROACH TO THE DESIGN OF AN AIRCRAFT GAIN SCHEDULED CONTROLLER

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M.S., The Royal Institute of Technology, 1985

Submitted in partial fulfillment of the
requirements for the degree of

AERONAUTICAL AND ASTRONAUTICAL ENGINEER

from the

NAVAL POSTGRADUATE SCHOOL

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ABSTRACT

This thesis addresses the problem of integrated design of the aircraft plant parameters and of the corresponding feedback controller. The plant parameters are typically the sizes of the control surfaces or other aerodynamical surfaces of the aircraft. The approach is to rewrite the aircraft dynamic requirements as linear matrix inequalities (LMI's) and to optimize a linear cost function associated with aircraft plant parameters, while meeting the LMI constraints. An algorithm using Matlab and LMI-Lab has been developed. This algorithm has been used for integrated plant/controller design of an F-4 aircraft at five different flight conditions. The result consists of a set of controllers, one for each flight condition, and a single solution for the optimal sizes of the F-4 stabilator and spoiler control surfaces.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ACKNOWLEDGMENT

I would like to thank my past and present superiors at the National Defence Research Establishment for letting me attend the Naval Postgraduate School. I would also like to extend my sympathies to the overburdened Swedish taxpayers for having to pay the high costs associated with this education. My advisor Professor Isaac Kaminer has been a good guide and companion on the winding path of science. I would like to thank Professor Richard Howard for his assistance in flight dynamics and Dr Pascal Gahinet for helping me to get LMI-Lab to work. Finally I would like to thank my wife Linda for making our stay in California more enjoyable and for letting me assist her with her studies and thereby giving me something useful to do in the spare time I had before I became overwhelmed by the work with this thesis.

I. INTRODUCTION

Control systems of modern aircraft have become increasingly complex. That is especially true in the case of fighter aircraft with relaxed stability and multiple control surfaces working in each axis.

The control system, control surfaces included, must satisfy all the performance requirements (stability, disturbance rejection, maneuverability etc.), while not causing excessive drag, weight or signatures. It is thus interesting to study the problem of finding the smallest size of some parameters describing the physical configuration of the aircraft, while still being able to find a controller that satisfies the performance requirements.

While the importance of integrated plant and controller design has been recognized, little had been done in the area prior to [Ref. 1]. In this thesis the ideas of integrating a minimization of the control surfaces with the design of a controller, that were developed in [Ref. 1], are developed further.

The design procedure is based on a set of requirements that are to be fulfilled at a number of different flight conditions and a cost function for the plant parameters, e.g. the size of the control surfaces. The result consists of a set of controllers, one for each flight condition, and one single optimal solution for the size of the plant parameters. The proposed design procedure is still in an early prototype phase, but shows promise of becoming a useful tool.

The thesis is organized as follows. Chapter II contains the formulation of the treated problem as an H_∞ control problem, and the parametrization of the plant. In Chapter III the algorithm from Chapter II is applied to the longitudinal dynamics of an F-4 fighter aircraft. The plant parameters are chosen to be the sizes of the

moving horizontal tail and of the spoiler. Chapter II also contains the results from the plant optimization and the associated controllers. The program code in Matlab and LMI-Lab that is used to solve this problem is included in the appendix.

II. PROBLEM STATEMENT

The task of designing the control system of an aircraft has become increasingly complex in recent time. That is especially true in the case of modern fighter aircraft with relaxed stability and many control surfaces working in each axis. The control system must, of course, meet all the imposed performance requirements, but as excessive control capability costs weight, space, money, signature etc., it is desireable to optimize the control system.

The problem to be addressed is to find the minimal size of some plant parameters (typically the size of the control surfaces or other aerodynamical surfaces), while finding a feedback controller that meets the stated control requirements. The design requirements are given for several flight conditions and the designed combination of plant parameter and controller must, of course, work for all of the flight conditions. The result of the optimization is one set of values for the plant parameters and a set of controllers, one for each flight condition. The objective is to use this set of controllers to design a gain scheduled controller, that works for all the given flight conditions (and hopefully in between those given conditions).

When the sizes of the surfaces are varied, the dynamics of the aircraft changes. It is therefore natural to integrate the minimization of a cost function associated with the plant parameters with the controller design. The controller is designed using H_∞ methods, which are summarized in Section A. The problem of minimizing the plant parameters, while still being able to find a feasible controller, is treated in Section B.

A. H_∞ CONTROL

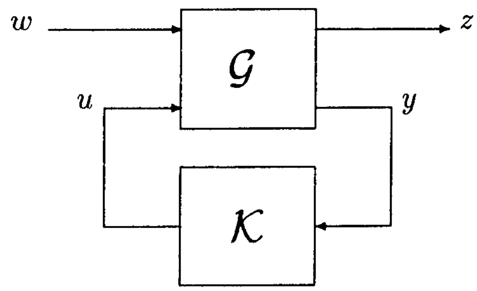

Figure 2.1: Feedback interconnection of plant $\mathcal{G}$ and controller $\mathcal{K}$.

A general feedback system with exogenous input w and output to be controlled z is shown in Figure 2.1. In this context we will only consider linear, time invariant systems. Here w is a reference signal that includes disturbance inputs. We wish the system to reject the disturbance and track the reference signal, i.e. we want a controller that makes the difference between the reference input and the actual output as small as possible, while not expending too much control energy. This is accomplished by making z, which is a combination of tracking errors and the control inputs, as small as possible. Let $T_{zw}(G, K)$ denote the closed loop transfer fuction from the input w to the output z. The solution to the H_∞ control problem is the controller K that minimizes the infinity norm of the transfer function, i.e. $\|T_{zw}(G, K)\|_\infty$.

The infinity norm is a generalization of the maximal gain and is defined as the supremum over all frequencies of the maximal singular value of the transfer function, i.e.

$$\|T_{zw}\|_\infty = \sup_\omega \{\overline{\sigma}(T_{zw}(j\omega))\}$$

Although the above definition is given in terms of the frequency domain, state-space methods are very useful in H_∞ synthesis. A state-space realization of the plant

depicted in Figure 2.1 with state feedback can be written as

$$\dot{x} = Ax + B_1 w + B_2 u \tag{2.1}$$

$$z = Cx + Du \tag{2.2}$$

$$y = x \tag{2.3}$$

Where $x \in R^n$, $z \in R^p$, $u \in R^m$ and $w \in R^q$.

To ensure the existence of a solution to the H_∞ control problem, we have to assume that (C_2, A, B_2) is stabilizable and detectable, and that D has linearly independant columns. The inputs and outputs, i.e. w and z, are usually scaled in such a way that the design requirements are satisfied if $\|T_{zw}(G,K)\|_\infty < 1$.

It can be shown [Ref. 2] that there exists a controller K such that $\|T_{zw}(G,K)\|_\infty < 1$ if and only if there exist W and Y, with Y symmetric and positive definite, such that

$$R(W,Y) = \begin{bmatrix} AY + YA^T + B_2W + W^TB_2^T & B_1 & (CY + DW)^T \\ B_1^T & -I & 0 \\ CY + DW & 0 & -I \end{bmatrix} < 0 \tag{2.4}$$

If the linear (or rather affine) matrix inequality (2.4) has a solution, the controller $K = WY^{-1}$ solves the H_∞ control problem.

B. PLANT PARAMETRIZATION

We consider a dynamical system at several given operating conditions, or more specifically an aircraft with performance requirements given for several flight conditions. At each of the flight conditions we use a linear model to describe the dynamics of the aircraft. The linear models are obtained by linearizing the general non-linear six degree of freedom equations of motion for each flight condition. The linearized models are different for each flight condition and with i, $i = 1, 2, ...l$, denoting the respective flight conditions we have

$$\dot{x} = A^i x + B_1^i w + B_2^i u \tag{2.5}$$

$$z = C^i x + D^i u \tag{2.6}$$

Where $x \in R^n$ is the state-vector representing perturbations around the trimmed condition, $w \in R^q$ is the external input, $u \in R^m$ is the control input and $z \in R^p$ is the ouput to be regulated.

We now parametrize the plant with the plant parameter $\zeta = [\zeta_1, \zeta_2, ..., \zeta_k]^T$, where ζ is normalized such that $0 < \zeta_j \leq 1$. Associated with ζ is a linear cost function $J = c^T\zeta$, which we wish to minimize. The A^i, B_1^i and B_2^i matrices, describing the plant are parametrized as the first terms of the Taylor expansion around the point $\zeta = 0$ as

$$A^i(\zeta) = A_0^i + \sum_{j=1}^{k} A_{1j}^i \zeta_j \tag{2.7}$$

$$B_1^i(\zeta) = B_{10}^i + \sum_{j=1}^{k} B_{1j}^i \zeta_j \tag{2.8}$$

$$B_2^i(\zeta) = B_{20}^i + \sum_{j=1}^{k} B_{2j}^i \zeta_j \tag{2.9}$$

where

$$A_{1j}^i = \left[\frac{\partial A^i}{\partial \zeta_j}\right]_{\zeta=0} \tag{2.10}$$

$$B_{1j}^i = \left[\frac{\partial B_1^i}{\partial \zeta_j}\right]_{\zeta=0} \tag{2.11}$$

$$B_{2j}^i = \left[\frac{\partial B_2^i}{\partial \zeta_j}\right]_{\zeta=0} \tag{2.12}$$

The C^i and D^i matrices could also depend on ζ, but that case is not considered here.

For each of the flight conditions we can find a steady state H_∞ controller by solving (2.4) for W^i and Y^i. However, the plant matrices now depend on ζ, hence

$$\mathrm{R}(\mathrm{W^i},\mathrm{Y^i},\zeta) = \begin{bmatrix} A^i(\zeta)Y^i + Y^iA^{iT}(\zeta) + B_2^i(\zeta)W^i + W^{iT}B_2^{iT}(\zeta) & B_1^i(\zeta) & (C^iY^i + D^iW^i)^T \\ B_1^{iT}(\zeta) & -I & 0 \\ C^iY^i + D^iW^i & 0 & -I \end{bmatrix} < 0 \tag{2.13}$$

Our problem is thus

$\min\ J = c^T\zeta$

s.t. $(Y^i, W^i, \zeta) \in \Phi$

where the feasible set Φ is defined as

$\Phi = \{W^i \in R^{m\times n}, Y^i \in R^{n\times n}, \zeta : Y^i > 0, Y^i = Y^{iT}, 0 < \zeta_j \leq 1, R(W^i, Y^i, \zeta) < 0\}$

C. ALGORITHM

Following the results in [Ref. 1] the optimization problem in W^i, Y^i and ζ is solved as follows. We observe that R, for a fixed ζ, is affine in the two remaining variables W^i and Y^i. We further observe that R, for fixed Y^i and W^i, is affine in the sole remaining variable ζ.

Hence the approach is to:

1. Fix $\zeta = \zeta_0$ and find W^i and $Y^i > 0$, $i = 1, 2, ..., l$ such that $R(W^i, Y^i, \zeta_0) < 0$

2. With W^i and Y^i from step 1, find the ζ with $\zeta_j > 0$ that minimizes $J = c^T\zeta$ s.t. $R(W^i, Y^i, \zeta) < 0$

3. Go to step 1. until exit criterion is satisfied.

The problem to be solved in step 1 is to solve l LMI's for the matrices W^i and Y^i. The l LMI's are, however, independent of each other and can be solved in sequence. The problem in step 2 is that of solving one single LMI for the minimal plant parameter ζ. Here all flight conditions must be treated simultaneously, as the same physical aircraft must be used at all the flight conditions.

The solution of LMI's is supported by software packages such as LMI-Lab [Ref. 6], which is used in the example given in Chapter III. While the approach to formulate the problem as two LMI's solved in sequence makes the calculations manageble, there is unfortunately no guarantee that the resulting algorithm will converge to a stationary point, [Ref. 3]. Neither is it clear whether or not the feasible set Φ is convex. Convexity is, of course, of utmost importance in every optimization problem, as when a convex function is minimized over a convex set, all minimas are global.

Chapter III describes the application of this algorithm to the longitudinal dynamics of an F-4 fighter aircraft.

III. LONGITUDINAL CONTROL OF F-4

The specific example concerns the control of the longitudinal dynamics of an F-4C fighter aircraft. The objective is to size the two control surfaces, elevator and spoiler, and to design a controller for elevator, spoiler and thrust for each of the different flight conditions. Five flight conditions, three subsonic and two supersonic, with data from [Ref. 4] are used; see tables 3.1 and 3.2.

The problem formulation is basically the same as in [Ref. 1], i.e. to minimize the sizes of the elevator and the spoiler, while finding a controller that satisfies a set of performance requirements for each flight condition. The main difference is that thrust control is included and that multiple flight conditions are treated. The numerical algorithm, which uses LMI-Lab, is completely different from the one used in [Ref. 1].

In section A the equations of motion are parametrized according to the chosen parameters, elevator and spoiler. Section B treats the performance requirements and how they are expressed in terms of H_∞ control. Sections C-E contain the results from the performed calculations.

Nomenclature:

X_a Aerodynamic derivative with respect to states or controls in X-direction

Z_a Aerodynamic derivative with respect to states or controls in Z-direction

M_a Aerodynamic derivative with respect to states or controls in M-direction

I_y Moment of inertia around the y-axis

m Mass

U Trimmed velocity in x-direction

α_0 Trimmed angle of attack

l Distance from center of gravity to elevator

δ_e Elevator deflection

δ_s Spoiler deflection

δ_t Thrust perturbation from equilibrium

σ_α Disturbance in angle of attack, α

σ_γ Disturbance in flight path angle, γ

A. EQUATIONS OF MOTION AND AERODYNAMICAL DATA

The equations describing the dynamics of an aircraft in six degrees of freedom consist of a set of coupled non-linear differential equations. These equations are conveniently formulated in a body fixed coordinate system. In the design of the control system, we generally consider the set of linearized equations describing the dynamics around a certain nominal flight condition.

In this section we use a model with four linear differential equations to describe the longitudinal dynamics. In general the longitudinal dynamics of an aircraft can be divided into two modes: the short period and the long period. From [Ref. 4] we obtain aerodynamical data for an F-4 fighter aircraft operating at five flight conditions: three subsonic and two supersonic.

The plant parameters to be optimized are the size of the moving horizontal tail and the size of a spoiler. These sizes are represented by the two coefficients Z_δ and Z_s, which give the vertical acceleration per radian of elevator and spoiler deflection respectively. Hence, the plant parameter is $\zeta = [\frac{Z_\delta}{Z_{\delta nominal}}, \frac{Z_s}{Z_{snominal}}]^T$. The aerodynamical data is adapted to fit that parametrization, by first calculating the aerodynamical derivatives for the condition $\zeta = 0$ and then calculating all the matrices needed in the equations of motion.

TABLE 3.1: Aircraft Data (Ref. 4)

Flight Condition	(1)	(2)	(3)	(4)	(5)
Altitude (ft)	15,000	35,000	35,000	35,000	45,000
Mach	0.9	0.6	0.9	1.2	1.5
$V(ft/s)$	952	584	876	1167	1452
$q(lb/ft^2)$	677	126	283	503	489
$m(slugs)$	1210	1210	1210	1210	1210
$I_x(slug \cdot ft^2)$	24970	27360	25040	24970	25040
$Iy_(slug \cdot ft^2)$	122190	122190	122190	122190	122190
$I_z(slug \cdot ft^2)$	13970	139800	13730	139800	139730
$I_{xz}(slug \cdot ft^2)$	1175	-16432	-3033	-1030	-3033
$\alpha_{Trim}(deg)$	0.5	9.4	2.6	1.6	2.6

The spoiler is an add-on to the data from [Ref. 4] and is modeled in the same manner as in [Ref. 1], i.e. as giving a maximal acceleration of $0.2g$ in the z-direction and a corresponding acceleration in the x-direction of $-0.02g$.

We need to find out how the varying of the plant parameters affect the aerodynamical derivatives describing the aircraft dynamics. Changing the size of the moving tail affects the dynamics of the aircraft drastically. As the tail is reduced the stability as well as the damping is reduced. The other plant parameter, the spoiler, does, however, not affect any coefficients other than X_s and Z_s. The division of the aerodynamic derivatives into one part that depends on the wing and body of the aircrft and one part that depends on the tail is done according to [Ref. 5] as

$$Z_{\alpha wb} = Z_\alpha - Z_\delta \tag{3.1}$$

$$Z_{qwb} = Z_q - \frac{l}{U} Z_\delta \tag{3.2}$$

$$Z_{\dot{\alpha} wb} = Z_{\dot{\alpha}} - \frac{l}{U} Z_\delta \tag{3.3}$$

$$M_{\alpha wb} = M_\alpha - \frac{lm}{I_y} Z_\delta \tag{3.4}$$

$$M_{qwb} = M_q - \frac{l^2 m}{I_y U} Z_\delta \tag{3.5}$$

TABLE 3.2: Nominal Dimensional Derivatives (Ref. 4)

Flight Condition	(1)	(2)	(3)	(4)	(5)
$X_u(s^{-1})$	-0.0215	-0.0176	-0.0123	-0.0136	-0.0072
$X_\alpha(ft/s^2)$	-5.93	-24.25	-7.43	-16.53	-29.98
$Z_u(s^{-1})$	-1.170	-0.337	-0.571	-0.747	-0.515
$Z_\alpha(ft/s^2)$	-1103.2	-162.2	-475.4	-848.3	-716.7
$Z_{\dot{\alpha}}(ft/s)$	-2.00	-0.591	-1.01	-1.24	-0.519
$Z_q(ft/s)$	-6.00	-1.82	-2.89	-4.09	-2.24
$M_u(ft^{-1}s^{-1})$	-0.0044	-0.0001	-0.0028	0.0022	0.0025
$M_\alpha(s^{-2})$	-17.00	-1.927	-7.887	-29.03	-28.94
$M_{\dot{\alpha}}(s^-1)$	-0.457	-0.141	-0.234	-0.288	-0.122
$M_q(s^{-1})$	-0.993	-0.307	-0.487	-0.746	-0.488
$X_\delta(ft/s^2)$	0.00	-0.01	0.00	-0.01	0.00
$Z_\delta(ft/s^2)$	-107.0	-20.98	-49.65	-90.44	-70.67
$M_\delta(s^{-2})$	-25.00	-4.90	-11.40	-20.70	-16.00
$X_s(ft/s^2)$	-0.919	-0.919	-0.919	-0.919	-0.919
$Z_s(ft/s^2)$	9.19	9.19	9.19	9.19	9.19

$$M_{\dot{\alpha}wb} \quad = \quad M_{\dot{\alpha}} - \frac{l^2 m}{I_y U} Z_\delta \tag{3.6}$$

The resulting wing/body aerodynamical derivatives are shown in Table 3.3. A comparison with the values in Table 3.2 shows that the removal of the horizontal tail causes the static stability to decrease. In the three subsonic conditions the aircraft is now unstable ($M_\alpha > 0$). The damping has also decreased, i.e. $M_{\dot{\alpha}} + M_q$ is now closer to zero. The fact that $M_{\dot{\alpha}}$ is positive is unusual, but as it is usually difficult to separate $M_{\dot{\alpha}}$ and M_q, and they still add up to a negative number, there is no need for concern.

For the longitudinal dynamics we use the linearized equations of motion [Ref. 4] with the four state variables given as $[u/U, \alpha, q, \theta]^T$. These states represent perturbations around the trimmed values $[U, \alpha_0, 0, \theta_0]^T$. By using the dimensional aerodynamic

TABLE 3.3: Wing/Body Dimensional Derivatives

Flight Condition	(1)	(2)	(3)	(4)	(5)
$X_u(s^{-1})$	-0.0215	-0.0176	-0.0123	-0.0136	-0.0072
$X_\alpha(ft/s^2)$	-5.93	-24.25	-7.43	-16.53	-29.98
$Z_u(s^{-1})$	-1.170	-0.337	-0.571	-0.747	-0.515
$Z_\alpha(ft/s^2)$	-996.2	-141.2	-425.75	-757.9	-646.0
$Z_{\dot\alpha}(ft/s)$	0.652	0.256	0.318	0.551	0.594
$Z_q(ft/s)$	-3.35	-0.973	-1.56	-2.30	-1.13
$M_u(ft^{-1}s^{-1})$	-0.0044	-0.0001	-0.0028	0.0022	0.0025
$M_\alpha(s^{-2})$	8.00	2.97	3.52	-8.33	-12.94
$M_{\dot\alpha}(s^-1)$	0.163	0.057	0.071	0.122	0.130
$M_q(s^{-1})$	-0.373	-0.109	-0.182	-0.336	-0.236

derivatives the equations of motion for the longitudinal case assume the form

$$I_n^i \dot{x} = A_n^i x + B_{1n}^i w + B_{2n}^i u \tag{3.7}$$

where $w = [\alpha_{ref}, \gamma_{ref}]^T$ is the external input and $u = [\delta_e, \delta_s, \delta_t]^T$ is the commanded values of elevator deflection, spoiler deflection and thrust.

With aerodynamical data for the respective flight condition, the plant matrices are given as

$$I_n^i = \begin{bmatrix} U & 0 & 0 & 0 \\ 0 & U - Z_{\dot\alpha} & 0 & 0 \\ 0 & -M_{\dot\alpha} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.8}$$

$$A_n^i = \begin{bmatrix} UX_u & X_\alpha & 0 & -g\cos\theta_0 \\ UZ_u & Z_\alpha & U + Z_q & -g\sin\theta_0 \\ UM_u & M_\alpha & M_q & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.9}$$

$$B_{1n}^i = \begin{bmatrix} X_\alpha & 0 \\ Z_\alpha & 0 \\ M_\alpha & 0 \\ 0 & 0 \end{bmatrix} \tag{3.10}$$

$$B_{2n}^i = \begin{bmatrix} 0 & X_s & 1 \\ Z_\delta & Z_s & 0 \\ M_\delta & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.11}$$

$$x = \begin{bmatrix} u/U \\ \alpha \\ q \\ \theta \end{bmatrix} \tag{3.12}$$

$$w = \begin{bmatrix} \alpha_{ref} \\ \gamma_{ref} \end{bmatrix} \tag{3.13}$$

$$u = \begin{bmatrix} \delta_e \\ \delta_s \\ \delta_t \end{bmatrix} \tag{3.14}$$

We now write the A^i, B_1^i, and B_2^i matrices as

$$A^i(\zeta) = A_0^i + \sum_{j=1}^{2} A_j^i \zeta_j \tag{3.15}$$

$$B_1^i(\zeta) = B_{10}^i + \sum_{j=1}^{2} B_{1j}^i \zeta_j \tag{3.16}$$

$$B_2^i(\zeta) = B_{20}^i + \sum_{j=1}^{2} B_{2j}^i \zeta_j \tag{3.17}$$

This is the equivalent of taking the first two terms in the Taylor series expansion around the point $\zeta = [0, 0]^T$. Using the matrices defined in (3.8-3.11) we have

$$A_0^i = (I_n^i)^{-1} A_n^i \tag{3.18}$$

$$A_1^i = \frac{\partial A_0^i}{\partial \zeta_1} = \frac{\partial A_0^i}{\partial Z_\delta} Z_{\delta nom} = (\frac{\partial (I_n^i)^{-1}}{\partial Z_\delta} A_n^i + (I_n^i)^{-1} \frac{\partial A_n^i}{\partial Z_\delta}) Z_{\delta nom} \tag{3.19}$$

$$A_2^i = \frac{\partial A_0^i}{\partial \zeta_2} = \frac{\partial A_0^i}{\partial Z_s} Z_{snom} = (\frac{\partial I_n^{i-1}}{\partial Z_s} A_n^i + I_n^{i-1} \frac{\partial A_n^i}{\partial Z_s}) Z_{snom} \tag{3.20}$$

$$B_{10}^i = (I_n^i)^{-1} B_{1n}^i \tag{3.21}$$

$$B_{11}^i = \frac{\partial B_{10}^i}{\partial \zeta_1} = \frac{\partial B_{10}^i}{\partial Z_\delta} Z_{\delta nom} = (\frac{\partial (I_n^i)^{-1}}{\partial Z_\delta} B_{1n}^i + (I_n^i)^{-1} \frac{\partial B_{1n}^i}{\partial Z_\delta}) Z_{\delta nom} \tag{3.22}$$

$$
\begin{aligned}
B_{12}^i &= \frac{\partial B_{10}^i}{\partial \zeta_2} = \frac{\partial B_{10}^i}{\partial Z_s} Z_{snom} = \left(\frac{\partial (I_n^i)^{-1}}{\partial Z_s} B_{1n}^i + (I_n^i)^{-1} \frac{\partial B_{1n}^i}{\partial Z_s}\right) Z_{snom} && (3.23)\\
B_{20}^i &= (I_n^i)^{-1} B_{2n}^i && (3.24)\\
B_{21}^i &= \frac{\partial B_{20}^i}{\partial \zeta_1} = \frac{\partial B_{20}^i}{\partial Z_\delta} Z_{\delta nom} = \left(\frac{\partial (I_n^i)^{-1}}{\partial Z_\delta} B_{2n}^i + (I_n^i)^{-1} \frac{\partial B_{2n}^i}{\partial Z_\delta}\right) Z_{\delta nom} && (3.25)\\
B_{22}^i &= \frac{\partial B_{20}^i}{\partial \zeta_2} = \frac{\partial B_{20}^i}{\partial Z_s} Z_{snom} = \left(\frac{\partial (I_n^i)^{-1}}{\partial Z_s} B_{2n}^i + (I_n^i)^{-1} \frac{\partial B_{2n}^i}{\partial Z_s}\right) Z_{snom} && (3.26)
\end{aligned}
$$

Where all the matrices are evaluated at $[Z_\delta, Z_s] = [0,0]$. We now calculate the needed derivatives.

$$
\frac{\partial (I_n^i)^{-1}}{\partial Z_\delta} = \frac{1}{U(U - Z_{\dot\alpha})^2} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & M_{\dot\alpha} + \frac{l^2 m}{I_y}(U - Z_{\dot\alpha}) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.27}
$$

$$
\frac{\partial (I_n^i)^{-1}}{\partial Z_s} = 0 \tag{3.28}
$$

$$
\frac{\partial A_n^i}{\partial Z_\delta} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1/U & 0 \\ 0 & \frac{lm}{I_y} & \frac{l^2 m}{I_y U} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.29}
$$

$$
\frac{\partial A_n^i}{\partial Z_s} = 0 \tag{3.30}
$$

$$
\frac{\partial B_{1n}^i}{\partial Z_\delta} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ \frac{lm}{I_y} & 0 \\ 0 & 0 \end{bmatrix} \tag{3.31}
$$

$$
\frac{\partial B_{1n}^i}{\partial Z_s} = 0 \tag{3.32}
$$

$$
\frac{\partial B_{2n}^i}{\partial Z_\delta} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ \frac{lm}{I_y} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.33}
$$

$$\frac{\partial B_{2n}^i}{\partial Z_s} = \begin{bmatrix} 0 & X_s/Z_s & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.34}$$

B. DESIGN OBJECTIVES AND AUGMENTED SYSTEM

The design objectives follow those outlined in [Ref. 1]:

- Step response - There should be no steady state error in the response to a step input in angle of attack, α, or flight path angle, $\gamma = \theta - \alpha$.
- Closed loop must be internally stable.
- Gust disturbaces with a magnitude of 5 ft/s in the z-direction should not yield elevator deflection greater than 20 degrees, spoiler deflection greater than 40 degrees or errors in angle of attack greater than 1.5 degrees.
- A flight path angle of 1 degree must be generated using elevator and spoiler deflection of less than 20 and 40 degrees, respectively.

To accommodate the steady state tracking requirement, the plant is augmented with the integrals of α and γ. The augmented system has six states, $x = [u/U, \alpha, q, \theta, \alpha/s, \gamma/s]^T$. The parametrized augmented system is thus

$$\begin{aligned} \dot{x_a} &= (A_{a0}^i + \sum_{j=1}^{2} A_{aj}^i \zeta_j)x_a + (B_{a10}^i + \sum_{j=1}^{2} B_{a1j}^i \zeta_j)w + (B_{a20}^i + \sum_{j+1}^{2} B_{a2j}^i \zeta_j)u \quad (3.35) \\ z &= C_a^i x_a + D_a^i u \quad (3.36) \end{aligned}$$

with the respective matrices given as

$$A_{a0}^i = \begin{bmatrix} & & & & 0 & 0 \\ & & A_0^i & & 0 & 0 \\ & & & & 0 & 0 \\ & & & & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{3.37}$$

$$A^i_{a1} = \begin{bmatrix} & & & & 0 & 0 \\ & & A^i_1 & & 0 & 0 \\ & & & & 0 & 0 \\ & & & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.38}$$

$$A^i_{a2} = 0 \tag{3.39}$$

$$B^i_{a10} = \begin{bmatrix} B^i_{10}\sigma_\alpha & \\ & \\ & \\ 0 & 0 \\ 0 & \sigma_\gamma \end{bmatrix} \tag{3.40}$$

$$B^i_{a11} = \begin{bmatrix} B^i_{11}\sigma_\alpha & \\ & \\ & \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{3.41}$$

$$B^i_{a12} = 0 \tag{3.42}$$

$$B^i_{a20} = 0 \tag{3.43}$$

$$B^i_{a20} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.44}$$

$$B^i_{a21} = \begin{bmatrix} & B^i_{21} & \\ & & \\ & & \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.45}$$

$$B_{a22}^i = \begin{bmatrix} & B_{22}^i & \\ & & \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.46}$$

$$C_a^i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\alpha_{max}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & w_\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & w_\gamma \end{bmatrix} \tag{3.47}$$

$$D_a^i = \begin{bmatrix} \frac{1}{\delta_{emax}} & 0 & 0 \\ 0 & \frac{1}{\delta_{smax}} & 0 \\ 0 & 0 & \frac{1}{\delta_{tmax}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.48}$$

$$z = \begin{bmatrix} \delta_e \\ \delta_s \\ \delta_t \\ \alpha \\ \alpha/s \\ \gamma/s \end{bmatrix} \tag{3.49}$$

The scaling of the in- and output to get $\|T_{zw}\|_\infty < 1$ shows up through the σ_α and σ_γ in the B_{a1} matrices and through the weights in the C_a and D_a matrices.

C. CALCULATIONS

The calculations were done in Matlab. The code is listed in the appendix. The linear matrix inequalities are solved by using LMI-Lab, [Ref. 6], which is a recently developed toolbox to Matlab. W^i and Y^i, i.e. the controllers, are obtained from the function "feasp" and the minimization of ζ is done by using "linobj". The function "feasp" sometimes, however, returns a solution that is not at all negative definite.

That problem is solved by testing the calculated solution and, when it is infeasible, discarding it in favor of the solution from the previous iteration. When "feasp" is again called in the next iteration, it usually gives a feasible solution.

The code is fairly fast - a run takes about 10 minutes on a Sparc 10.

The calculations were performed with the following values:

$$c = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\sigma_\gamma = 1\,degree$$

$$\sigma_\alpha = 5/U_i$$

$$\delta_{emax} = 20\,degrees$$

$$\delta_{smax} = 40\,degrees$$

$$\delta_{tmax} = 0.05g$$

D. OPTIMIZATION RESULTS

As we are uncertain about whether or not the feasible set Φ is convex, the convergence properties of the algorithm were tested to see if the solution would converge to a single optimal point or not.

The convergence properties were tested by running the algorithm with different initial conditions. First we varied the initial value of the elevator, i.e. $\zeta_{1initial}$, between 0.1 and 1.5, while holding the initial value of the spoiler as the nominal, i.e. $\zeta_{2initial} = 1$. In 3.1 we see the optimization history for each iteration of the two plant parameters. The solution tends to converge to smaller values, the smaller the initial value is. The same results are shown in 3.2, where the two values ζ_1 and ζ_2 are plotted against each other. The cost function for the different initial values of ζ_1 is shown in Figure 3.3. It is obvious that the algorithm does not converge to the same solution, when it is run with different initial values.

The initial value of the spoiler, ie ζ_2, was varied between 1.5 and 0.6, while the initial value of the elevator was held constant at 0.5. The optimization history is shown in Figure 3.4, while Figure 3.5 shows ζ_2 as function of ζ_1. In this case the algorithm converges to the solution $\zeta_{opt} = [0.06, 0.55]^T$, for all initial conditions but the two smallest. Figure 3.6 shows the cost function for different initial values of ζ_2.

The conclusion is that the calculated solution is sensitive to the initial value, especially the initial value of ζ_1. The obvious conjecture is that the feasible set Φ not is convex.

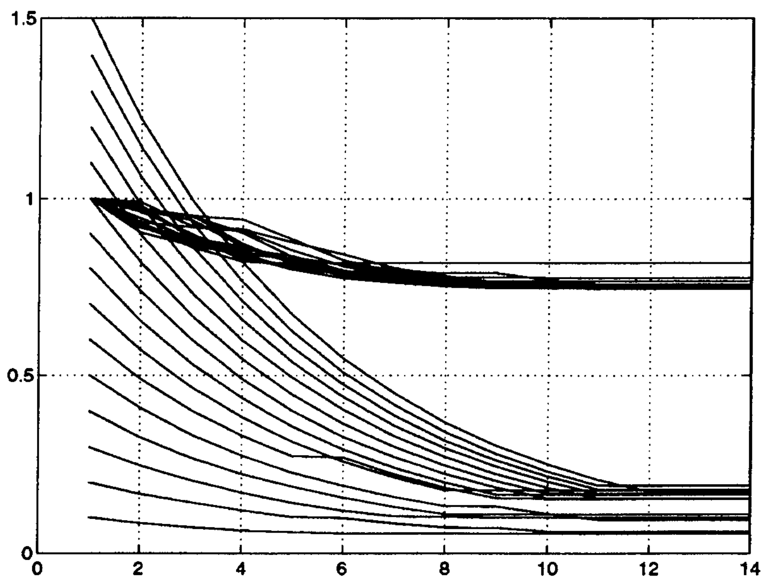

Figure 3.1: Optimization history of ζ - varying $\zeta_{1initial}$

The calculated solution always gives a very small size of the elevator. That is, however, consistent with the problem formulation.

E. CONTROLLERS

The controllers for the five flight conditions that are listed below, were calculated for $\zeta = [0.1, 0.7]^T$ which is close to the lowest values given in Figures (3.1-3.5). The resulting five controllers are presented in below. Tables (3.4-3.8) show the eigenvalues

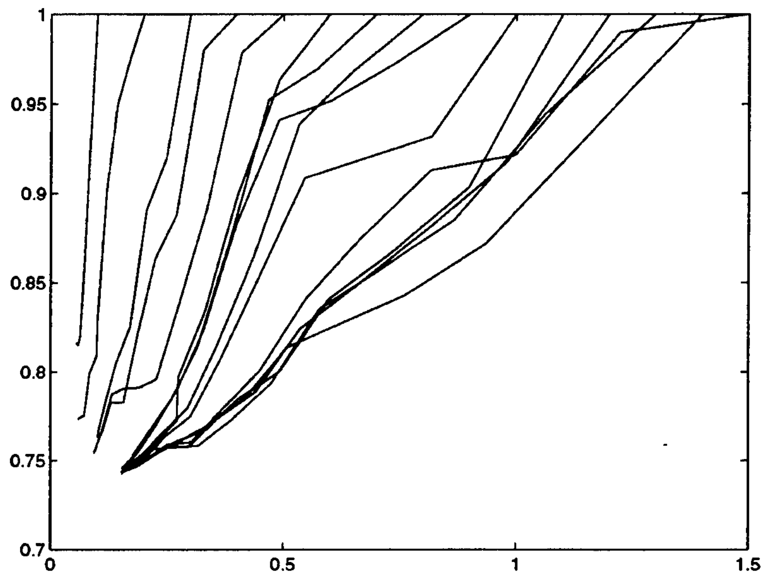

Figure 3.2: Optimization history: ζ_2 as function of ζ_1

and dampings for the closed loop system for the five flight conditions. The designed controllers give the closed loop system rather slow dynamics. This can be corrected by adjusting the weights in the C matrix. The damping, which varies between 0.54 and 0.88 is, however, good. As the purpose of the performed work is to develop a procedure for the design of an controller, rather than actually designing a controller, the results are clearly acceptable.

$$K^1 = \begin{bmatrix} 39.7 & -10.7 & 4.2 & 29.9 & 26.0 & 10.0 \\ 505.3 & -149.3 & 0.9 & 150.5 & -130.5 & 32.7 \\ -1201.0 & 297.1 & -1.7 & -300.7 & 258.9 & -65.1 \end{bmatrix}$$

$$K^2 = \begin{bmatrix} -3.76 & 36.86 & 17.07 & 5.77 & 57.94 & 2.70 \\ 105.6 & -148.4 & -2.04 & 139.9 & -39.57 & 31.30 \\ -154.97 & 71.75 & -0.246 & -72.83 & 4.70 & -17.23 \end{bmatrix}$$

$$K^3 = \begin{bmatrix} 15.28 & -2.55 & 7.49 & 27.44 & 40.65 & 7.74 \\ 230.2 & -151.4 & 0.913 & 151.9 & -59.50 & 32.15 \\ -362.5 & 153.8 & -1.101 & -156.1 & 55.94 & -31.67 \end{bmatrix}$$

$$K^4 = \begin{bmatrix} 38.51 & -19.58 & 3.80 & 33.03 & 29.71 & 8.88 \\ 459.4 & -213.9 & 1.06 & 215.3 & -100.1 & 41.96 \\ -837.7 & 295.6 & -1.44 & -298.4 & 134.5 & -57.38 \end{bmatrix}$$

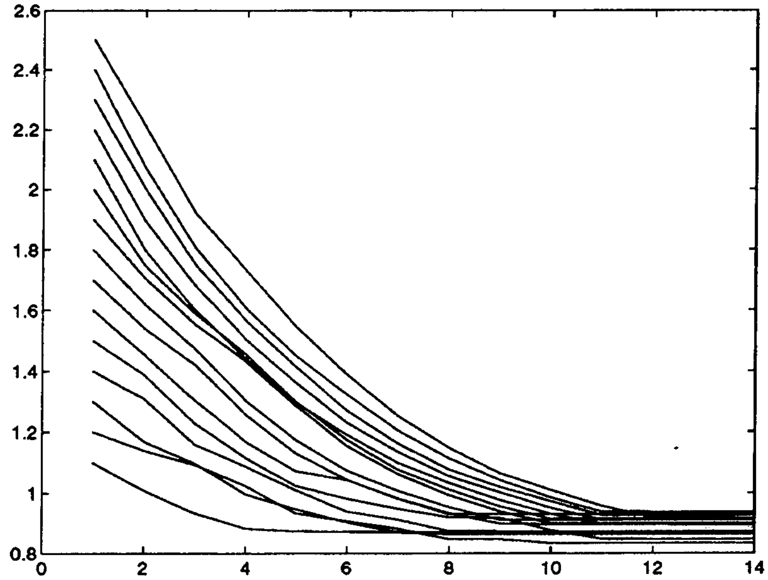

Figure 3.3: Costfunction for different $\zeta_{1initial}$ as function of ζ_1

$$K^5 = \begin{bmatrix} 105.4 & -64.3 & 6.8 & 87.6 & 23.3 & 17.5 \\ 787.0 & -505.4 & 2.4 & 510.5 & -149.9 & 89.2 \\ -1065.3 & 543.6 & -2.6 & -550.0 & 153.4 & -95.0 \end{bmatrix}$$

TABLE 3.4: Closed loop eigenvalues and damping, condition 1

Eigenvalue	Damping	Freq. (rad/sec)
-0.2343 + 0.1751i	0.8010	0.2924
-0.2343 - 0.1751i	0.8010	0.2924
-2.1488	1.0000	2.1488
-2.6423	1.0000	2.6423
-5.2115 + 4.9987i	0.7217	7.2213
-5.2115 - 4.9987i	0.7217	7.2213

Figures (3.7-3.9) show the actuator step response for flight condition 1, while figures (3.10-3.12) show the Bode plots for the actuator dynamics.

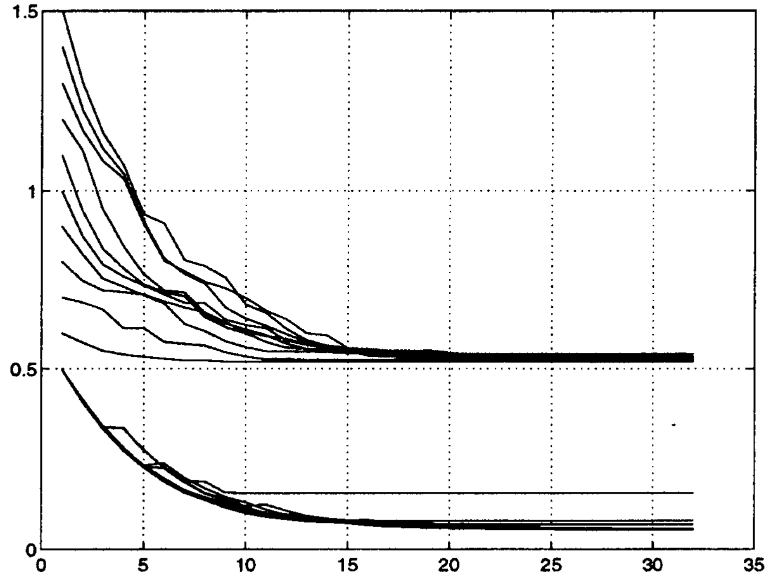

Figure 3.4: Optimization history of ζ - varying $\zeta_{2initial}$

TABLE 3.5: Closed loop eigenvalues and damping, condition 2

Eigenvalue	Damping	Freq. (rad/sec)
-0.2152 + 0.1162i	0.8799	0.2446
-0.2152 - 0.1162i	0.8799	0.2446
-1.4722	1.0000	1.4722
-1.8325 + 1.3402i	0.8072	2.2703
-1.8325 - 1.3402i	0.8072	2.2703
-5.7166	1.0000	5.7166

TABLE 3.6: Closed loop eigenvalues and damping, condition 3

Eigenvalue	Damping	Freq. (rad/sec)
-0.1905 + 0.1440i	0.7977	0.2388
-0.1905 - 0.1440i	0.7977	0.2388
-1.3747	1.0000	1.3747
-2.5642	1.0000	2.5642
-3.6290 + 3.1582i	0.7543	4.8108
-3.6290 - 3.1582i	0.7543	4.8108

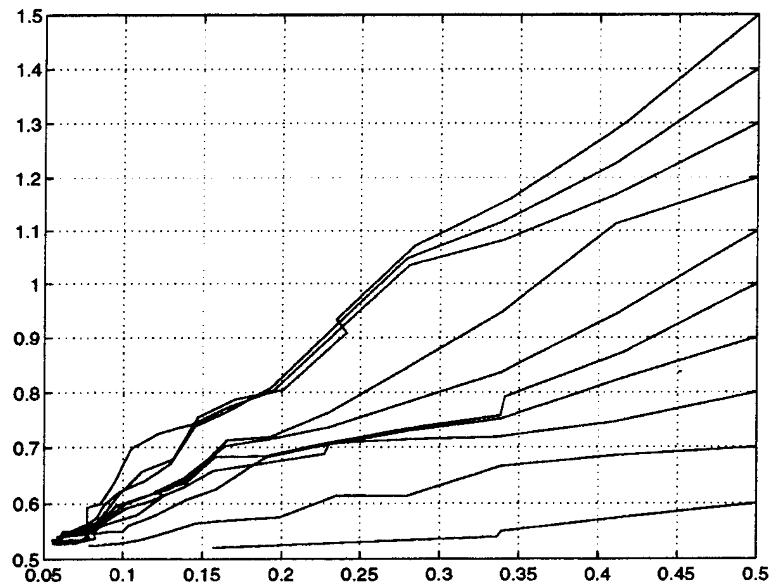

Figure 3.5: Optimization history: ζ_2 as function of ζ_1

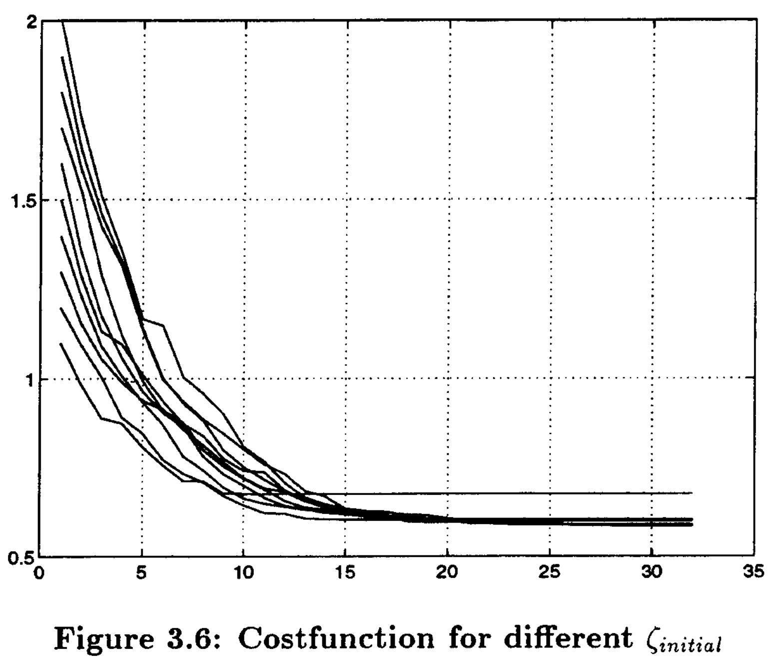

Figure 3.6: Costfunction for different $\zeta_{initial}$

TABLE 3.7: Closed loop eigenvalues and damping, condition 4

Eigenvalue	Damping	Freq. (rad/sec)
-0.1972 + 0.1466i	0.8026	0.2458
-0.1972 - 0.1466i	0.8026	0.2458
-1.6795	1.0000	1.6795
-2.1119	1.0000	2.1119
-3.7694 + 5.8935i	0.5388	6.9959
-3.7694 - 5.8935i	0.5388	6.9959

TABLE 3.8: Closed loop eigenvalues and damping, condition 5

Eigenvalue	Damping	Freq. (rad/sec)
-0.1697 + 0.1127i	0.8330	0.2038
-0.1697 - 0.1127i	0.8330	0.2038
-2.0966	1.0000	2.0966
-3.4113	1.0000	3.4113
-4.3355 + 3.9329i	0.7407	5.8536
-4.3355 - 3.9329i	0.7407	5.8536

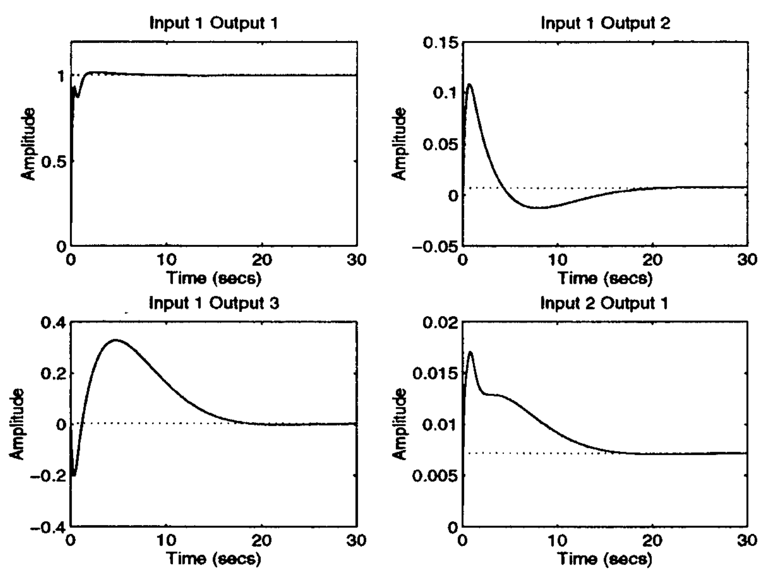

Figure 3.7: Actuator step response

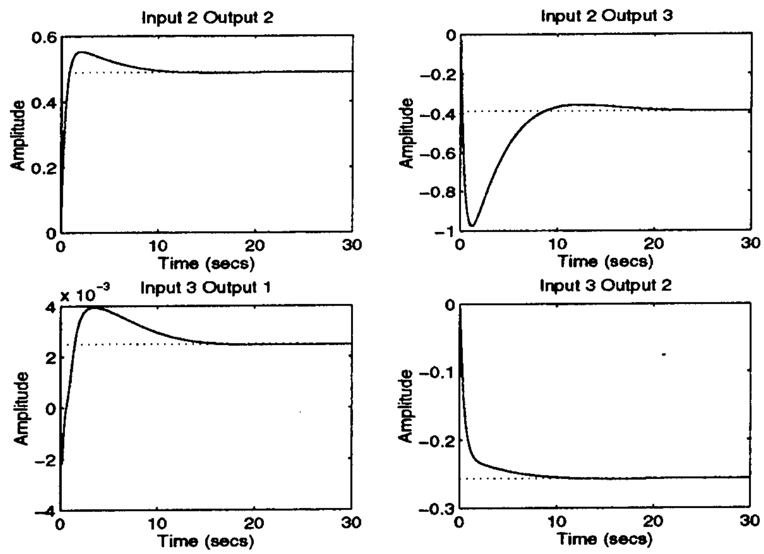

Figure 3.8: Actuator step response

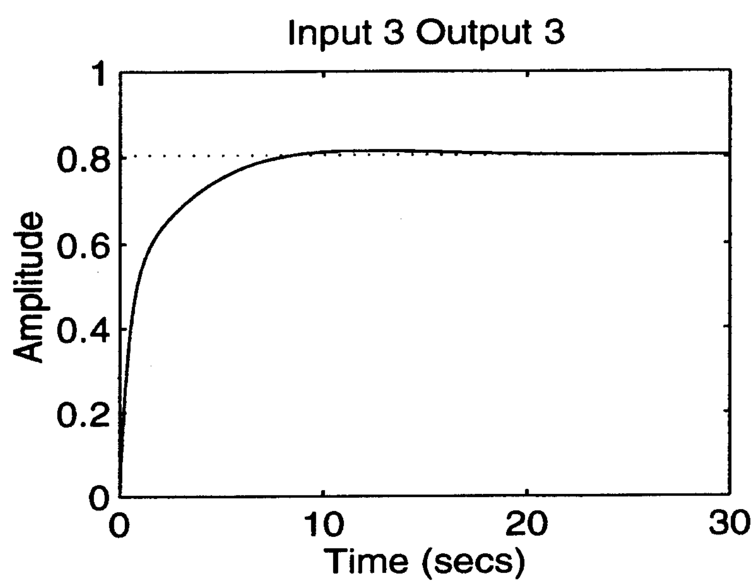

Figure 3.9: Actuator step response

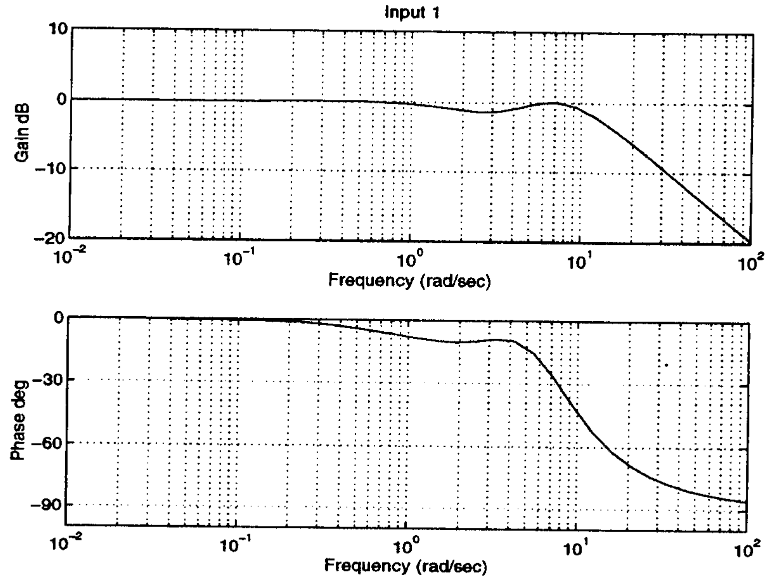

Figure 3.10: Bode plot for elevator

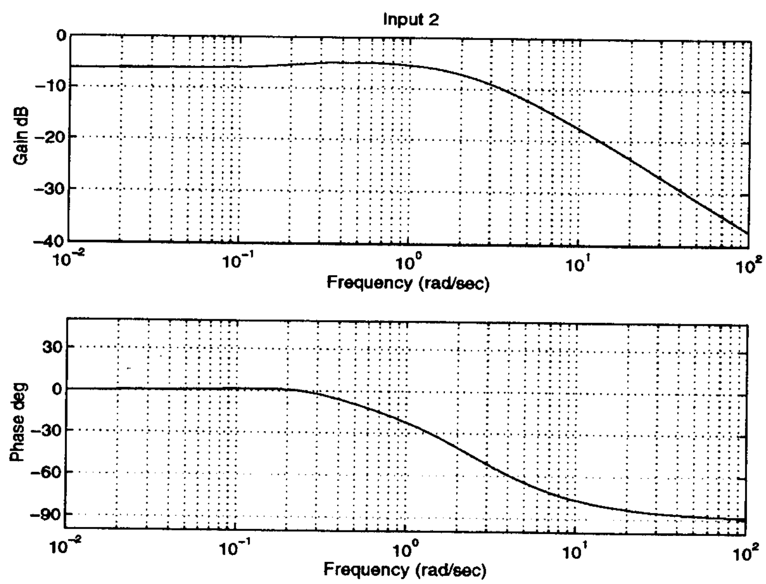

Figure 3.11: Bode plot for spoiler

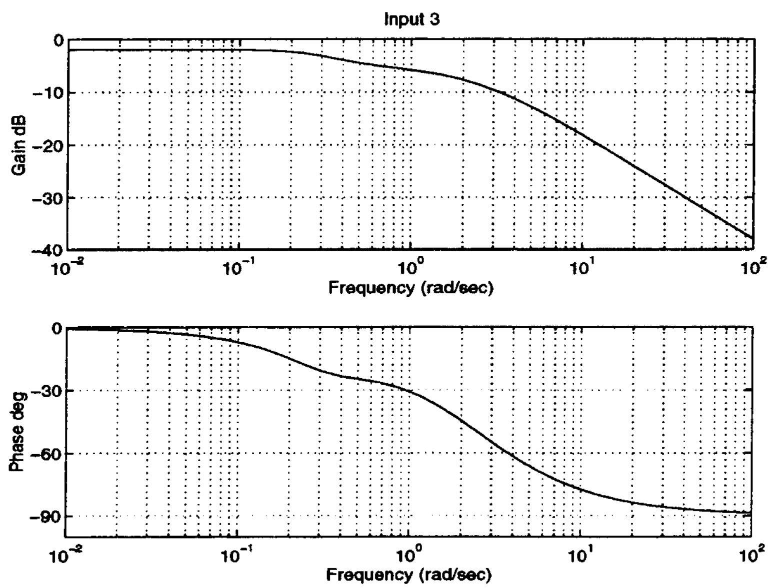

Figure 3.12: Bode plot for engine thrust

IV. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

This thesis develops the technique of integrating the design of aircraft plant parameters and the design of the corresponding feedback controller. The developed algorithm using Matlab and LMI-Lab gives a solution to the problem of minimizing the control surfaces of an aircraft, while finding a feasible H_∞ controller. The result, when this algorithm is applied to the longitudinal dynamics of an F-4 aircraft, is a set of controllers, one for each flight condition, and a single value of the sizes of the moving horizontal tail and the spoiler.

The approach used to formulate an integrated plant optimization and H_∞ controller design as a linear matrix inequality results in a non-convex optimization problem.

Compared to [Ref. 1] the problem formulation has been expanded to handle multiple flight conditions and to include thrust control. As the developed algorithm using LMI-Lab is more computationally efficient than the previous algorithm, it is possible to handle much larger problems than in [Ref. 1].

The calculated solutions do, however, depend on the initial value, as the problem is non-convex. It is thus necessary to run the algorithm several times, to be reasonably sure that a good quality solution has been obtained.

This work has increased the knowledge on how integrated plant optimization and controller design problems should be formulated and provides a good ground for further work. The completion of the recommended future work in the next section will make this approach to integrated plat/controller design a useful tool for engineers.

B. RECOMMENDATIONS

The convergence properties of the optimization algorithm are unclear and should be investigated. The first issue to consider is whether the algorithm converges to the global minimum for a convex problem. The second issue is to investigate the possibilities of finding a convex hull to the feasible set of plant parameters and controllers, and establish a lower bound for the optimal solution.

The problem treated in this thesis only concerns the dynamics of the aircraft for small perturbations around a trimmed equilibrium. The formulation does not include the fact that the aircraft configuration given by the optimal value of ζ must be possible to trim at the given flight condition. The problem formulation must be changed to include the requirement that the aircrft must be possible to trim.

The performance requirements concerning maneuverability are translated to an H_∞ formulation based on the linearized model developed for small perturbations around an equilibrium. A non-linear dynamical model should instead be used for the maneurability requirements.

The result of this work is a set of time invariant feedback controllers, each of them valid for one flight condition. This set of controllers can be used in the construction of a gain scheduled controller, that works for the non-linear aircraft dynamics at all flight conditions. The resulting closed loop system is time variant and non-linear. The gain scheduling must be done such that when this system is linearized at different flight conditions, the linear system has the desired properties. The linearized system dynamics should be as independent of the gain scheduling parameter as possible. A continuation of the work presented in this thesis is to find a way to interpolate between the different controller such that this is achieved.

APPENDIX A: MATLAB CODE

In the appendix the Matlab code using LMI-Lab is listed. The main program is f4opt.m, which uses the two functions sol4.m and sol5.m. This combination of Matlab and LMI-Lab needs to be run on a SUN Sparc station and does not work on other computers.

In f4opt.m the LMI's are formulated using two equivalent versions of the LMI

$$R(W^i, Y^i, \zeta) = \begin{bmatrix} A^i(\zeta)Y^i + Y^iA^{iT}(\zeta) + B_2^i(\zeta)W^i + W^{iT}B_2^{iT}(\zeta) & B_1^i(\zeta) & (C^iY^i + D^iW^i)^T \\ B_1^{iT}(\zeta) & -I & 0 \\ C^iY^i + D^iW^i & 0 & -I \end{bmatrix} < 0$$

these are

$$R1(W^i, Y^i, \zeta) = \begin{bmatrix} \begin{array}{c} A^i(\zeta)Y^i + Y^iA^{iT}(\zeta) + B_2^i(\zeta)W^i + W^{iT}B_2^{iT}(\zeta)+ \\ +B_1^i(\zeta)B_1^{iT}(\zeta) \end{array} & (C^iY^i + D^iW^i)^T \\ C^iY^i + D^iW^i & -I \end{bmatrix} < 0$$

that is used for solving for the controller, i.e. W and Y, and

$$R2(W^i, Y^i, \zeta) = \begin{bmatrix} \begin{array}{c} A^i(\zeta)Y^i + Y^iA^{iT}(\zeta) + B_2^i(\zeta)W^i + W^{iT}B_2^{iT}(\zeta)+ \\ +(C^iY^i + D^iW^i)^T(C^iY^i + D^iW^i) \end{array} & B_1^i(\zeta) \\ B_1^i(\zeta)^T & -I \end{bmatrix} < 0$$

that is used to solve for the optimal ζ.

```
% This program performs an integrated minimization of elevator and spoiler,
% and calculation of H-infinity controller. The aircraft is an F-4 with
% aircraft data from Schmidt. The calculations are performed for five
% different flight conditions.
% The program uses the toolbox to be LMI-Lab and the two matlab functions
% sol4 and sol5.
% The system is modeled as
% x_dot=A*x+B_1*w+B_2*u
% z=C*x+D_2*u
% The program calculates the H-infinity controller K=W*inv(Y) (u=K*x)
% To use this program to design a controller the weights in the B_1, B_2, C and
% D_2 matrices can be changed.
% The output of the program is stored in the arrays lam (maximum eigen value
% for each LMI), zet (plant parameters) and cost (cost function).

clear;

g=32.174; % ft/s^2
m=1210; % slugs
c=16.0; % ft
S=530; % ft^2
Xdlc=-0.02*g*57.1/40; % drag from spoiler
Zdlc=0.2*g*57.1/40; % destroyed lift from spoiler
c=[1;1]; % weights in cost function

% ****** Flight condition 6 ******

Iy=122190; % slug*ft
U=1452; % ft/s
Xu=-0.0072; % dimensional aerodynamic derivatives
Xa=-29.98;
Zu=-0.515;
Za=-716.7;
Zad=-0.519;
Zq=-2.24;
Mu=0.0025;
Ma=-28.94;
Mad=-0.122;
Mq=-0.488;
Xd=0.00;
Zd=-70.67;
Md=-16.00;
xc=Md/Zd*Iy/m; % lever for elevator
alpha0=2.6/57.3; % angle of attack in radians
theta0=alpha0;
```

```
thrust_acc=0.05*g; % max acceleration from thrust

sigma_g=1/57.3; % max commanded flight path angle
sigma_a=5/U; % wind gusts
umax1=20/57.3; % max elevator deflection
umax2=40/57.3; % max spoiler deflection
sigma_aout=1.5/57.3; % max error in alpha

C=[zeros(3,6);0 1/sigma_aout 0 0 0 0;0 0 0 0 60 0;0 0 0 0 0 10];

D2=[1/umax1 0 0;0 1/umax2 0;0 0 1/thrust_acc;zeros(3,3)];

% A matrix for nominal plant
An=[U*Xu Xa 0 -g*cos(theta0); U*Zu Za U+Zq -g*sin(theta0);
    U*Mu Ma Mq 0; 0 0 1 0];
In=[U 0 0 0;0 U-Zad 0 0;0 -Mad 1 0; 0 0 0 1];
A006=[inv(In)*An zeros(4,2);0 1 0 0 0 0;0 -1 0 1 0 0];

% Calculation of derivatives for wing/body
Za=Za-Zd;
Ma=Ma-xc*Zd*m/Iy;
Zq=Zq-xc*Zd/U;
Mq=Mq-xc*xc*Zd*m/Iy/U;
Zad=Zad-xc*Zd/U;
Mad=Mad-xc*xc*Zd*m/Iy/U;

An=[U*Xu Xa 0 -g*cos(theta0); U*Zu Za U+Zq -g*sin(theta0);
   U*Mu Ma Mq 0; 0 0 1 0];
invIn=[1/U 0 0 0;0 1/(U-Zad) 0 0;0 Mad/(U-Zad) 1 0; 0 0 0 1];
A06=[invIn*An zeros(4,2);0 1 0 0 0 0;0 -1 0 1 0 0];

dInd=[0 0 0 0;0 1 0 0; 0 Mad+xc*xc*m/Iy*(U-Zad) 0 0;0 0 0 0]/U/(U-Zad)^2;
dAnd=[0 0 0 0;0 1 1/U 0; 0 xc*m/Iy xc*xc*m/Iy/U 0;0 0 0 0];

A16=[(dInd*An+inv(In)*dAnd)*Zd zeros(4,2); zeros(2,6)];


B106=zeros(6,2);  B116=B106;  B126=B106;
B106(1:4,1)=A06(1:4,2)*sigma_a;
B106(6,2)=sigma_g;
dB106d=[0 0;1 0;xc*m/Iy 0;0 0]*sigma_a;
B116=[(dInd*B106(1:4,:)+invIn*dB106d)*Zd;zeros(2,2)];

B206=[0 0 1/U;zeros(5,3)];
dB206d=[0 0 0;1 0 0;xc*m/Iy 0 0;0 0 0];
```

```
dB206s=[0 Xdlc/Zdlc 0;0 1 0;0 0 0;0 0 0];
B216=[(dInd*B206(1:4,:)+invIn*dB206d)*Zd; zeros(2,3)];
B226=[invIn*dB206s*Zdlc; zeros(2,3)];
%************ Flight condition 5 ************************

U=1167;
Xu=-0.0136;
Xa=-16.53;
Zu=-0.747;
Za=-848.3;
Zad=-1.24;
Zq=-4.09;
Mu=0.0022;
Ma=-29.03;
Mad=-0.288;
Mq=-0.746;
Xd=0.00;
Zd=-90.44;
Md=-20.70;
xc=Md/Zd*Iy/m;
alpha0=1.6/57.3;
theta0=alpha0;
sigma_a=5/U;

An=[U*Xu Xa 0 -g*cos(theta0); U*Zu Za U+Zq -g*sin(theta0);
   U*Mu Ma Mq 0; 0 0 1 0];
invIn=[1/U 0 0 0;0 1/(U-Zad) 0 0;0 Mad/(U-Zad) 1 0; 0 0 0 1];
A05=[invIn*An zeros(4,2);0 1 0 0 0 0;0 -1 0 1 0 0];

dInd=[0 0 0 0;0 1 0 0; 0 Mad+xc*xc*m/Iy*(U-Zad) 0 0;0 0 0 0]/U/(U-Zad)^2;
dAnd=[0 0 0 0;0 1 1/U 0; 0 xc*m/Iy xc*xc*m/Iy/U 0;0 0 0 0];

A15=[(dInd*An+invIn*dAnd)*Zd zeros(4,2); zeros(2,6)];

B105=zeros(6,2);  B115=B105;  B125=B105;
B105(1:4,1)=A05(1:4,2)*sigma_a;
B105(6,2)=sigma_g;
dB105d=[0 0;1 0;xc*m/Iy 0;0 0]*sigma_a;
B115=[(dInd*B105(1:4,:)+invIn*dB105d)*Zd;zeros(2,2)];

B205=[0 0 1/U;zeros(5,3)];
dB205d=[0 0 0;1 0 0;xc*m/Iy 0 0;0 0 0];
dB205s=[0 Xdlc/Zdlc 0;0 1 0;0 0 0;0 0 0];
B215=[(dInd*B205(1:4,:)+invIn*dB205d)*Zd; zeros(2,3)];
B225=[invIn*dB205s*Zdlc; zeros(2,3)];
```

```
%***************** Flight condition 4 ****************************

U=867;
Xu=-0.0123;
Xa=-7.43;
Zu=-0.571;
Za=-475.4;
Zad=-1.01;
Zq=-2.89;
Mu=-0.0028;
Ma=-7.877;
Mad=-0.234;
Mq=-0.487;
Xd=0.00;
Zd=-49.65;
Md=-11.40;
xc=Md/Zd*Iy/m;
alpha0=2.6/57.3;
theta0=alpha0;
sigma_a=5/U;


An=[U*Xu Xa 0 -g*cos(theta0); U*Zu Za U+Zq -g*sin(theta0);
   U*Mu Ma Mq 0; 0 0 1 0];
invIn=[1/U 0 0 0;0 1/(U-Zad) 0 0;0 Mad/(U-Zad) 1 0; 0 0 0 1];
A04=[invIn*An zeros(4,2);0 1 0 0 0 0;0 -1 0 1 0 0];

dInd=[0 0 0 0;0 1 0 0; 0 Mad+xc*xc*m/Iy*(U-Zad) 0 0;0 0 0 0]/U/(U-Zad)^2;
dAnd=[0 0 0 0;0 1 1/U 0; 0 xc*m/Iy xc*xc*m/Iy/U 0;0 0 0 0];

A14=[(dInd*An+invIn*dAnd)*Zd zeros(4,2); zeros(2,6)];

B104=zeros(6,2);  B114=B104;  B124=B104;
B104(1:4,1)=A04(1:4,2)*sigma_a;
B104(6,2)=sigma_g;
dB104d=[0 0;1 0;xc*m/Iy 0;0 0]*sigma_a;
B114=[(dInd*B104(1:4,:)+invIn*dB104d)*Zd;zeros(2,2)];

B204=[0 0 1/U;zeros(5,3)];
dB204d=[0 0 0;1 0 0;xc*m/Iy 0 0;0 0 0];
dB204s=[0 Xdlc/Zdlc 0;0 1 0;0 0 0;0 0 0];
B214=[(dInd*B204(1:4,:)+invIn*dB204d)*Zd; zeros(2,3)];
B224=[invIn*dB204s*Zdlc; zeros(2,3)];

%************* Flight condition 3 ****************************
```

```
U=584;
Xu=-0.0176;
Xa=-24.25;
Zu=-0.337;
Za=-162.2;
Zad=-0.591;
Zq=-1.82;
Mu=-0.0001;
Ma=-1.927;
Mad=-0.141;
Mq=-0.307;
Xd=0.00;
Zd=-20.98;
Md=-4.90;
xc=Md/Zd*Iy/m;
alpha0=9.4/57.3;
theta0=alpha0;
sigma_a=5/U;

An=[U*Xu Xa 0 -g*cos(theta0); U*Zu Za U+Zq -g*sin(theta0);
   U*Mu Ma Mq 0; 0 0 1 0];
invIn=[1/U 0 0 0;0 1/(U-Zad) 0 0;0 Mad/(U-Zad) 1 0; 0 0 0 1];
A03=[invIn*An zeros(4,2);0 1 0 0 0 0;0 -1 0 1 0 0];

dInd=[0 0 0 0;0 1 0 0; 0 Mad+xc*xc*m/Iy*(U-Zad) 0 0;0 0 0 0]/U/(U-Zad)^2;
dAnd=[0 0 0 0;0 1 1/U 0; 0 xc*m/Iy xc*xc*m/Iy/U 0;0 0 0 0];

A13=[(dInd*An+invIn*dAnd)*Zd zeros(4,2); zeros(2,6)];

B103=zeros(6,2);  B113=B103;  B123=B103;
B103(1:4,1)=A04(1:4,2)*sigma_a;
B103(6,2)=sigma_g;
dB103d=[0 0;1 0;xc*m/Iy 0;0 0]*sigma_a;
B113=[(dInd*B103(1:4,:)+invIn*dB103d)*Zd;zeros(2,2)];

B203=[0 0 1/U;zeros(5,3)];
dB203d=[0 0 0;1 0 0;xc*m/Iy 0 0;0 0 0];
dB203s=[0 Xdlc/Zdlc 0;0 1 0;0 0 0;0 0 0];
B213=[(dInd*B203(1:4,:)+invIn*dB203d)*Zd; zeros(2,3)];
B223=[invIn*dB203s*Zdlc; zeros(2,3)];


%****************** Flight condition 2 **********************
```

```
U=952;
Xu=-0.0215;
Xa=-5.93;
Zu=-1.170;
Za=-1103.2;
Zad=-2.00;
Zq=-6.00;
Mu=-0.0044;
Ma=-17.00;
Mad=-0.457;
Mq=-0.993;
Xd=0.00;
Zd=-107.0;
Md=-25.00;
xc=Md/Zd*Iy/m;
alpha0=0.5/57.3;
theta0=alpha0;
sigma_a=5/U;

An=[U*Xu Xa 0 -g*cos(theta0); U*Zu Za U+Zq -g*sin(theta0);
   U*Mu Ma Mq 0; 0 0 1 0];
invIn=[1/U 0 0 0;0 1/(U-Zad) 0 0;0 Mad/(U-Zad) 1 0; 0 0 0 1];
A02=[invIn*An zeros(4,2);0 1 0 0 0 0;0 -1 0 1 0 0];

dInd=[0 0 0 0;0 1 0 0; 0 Mad+xc*xc*m/Iy*(U-Zad) 0 0;0 0 0 0]/U/(U-Zad)^2;
dAnd=[0 0 0 0;0 1 1/U 0; 0 xc*m/Iy xc*xc*m/Iy/U 0;0 0 0 0];

A12=[(dInd*An+invIn*dAnd)*Zd zeros(4,2); zeros(2,6)];

B102=zeros(6,2);  B112=B102;  B122=B102;
B102(1:4,1)=A02(1:4,2)*sigma_a;
B102(6,2)=sigma_g;
dB102d=[0 0;1 0;xc*m/Iy 0;0 0]*sigma_a;
B112=[(dInd*B102(1:4,:)+invIn*dB102d)*Zd;zeros(2,2)];

B202=[0 0 1/U;zeros(5,3)];
dB202d=[0 0 0;1 0 0;xc*m/Iy 0 0;0 0 0];
dB202s=[0 Xdlc/Zdlc 0;0 1 0;0 0 0;0 0 0];
B212=[(dInd*B202(1:4,:)+invIn*dB202d)*Zd; zeros(2,3)];
B222=[invIn*dB202s*Zdlc; zeros(2,3)];
%*****************************************************************
% Calculations
%*****************************************************************
% Inital value of zeta
```

```
cost=[];
zet=[];

zeta=[0.1;0.7];
 costold=c*zeta;

for iter=1:40

A2=A02+A12*zeta(1);
B12=B102+B112*zeta(1);
B22=B202+B212*zeta(1)+B222*zeta(2);

A3=A03+A13*zeta(1);
B13=B103+B113*zeta(1);
B23=B203+B213*zeta(1)+B223*zeta(2);

A4=A04+A14*zeta(1);
B14=B104+B114*zeta(1);
B24=B204+B214*zeta(1)+B224*zeta(2);

A5=A05+A15*zeta(1);
B15=B105+B115*zeta(1);
B25=B205+B215*zeta(1)+B225*zeta(2);

A6=A06+A16*zeta(1);
B16=B106+B116*zeta(1);
B26=B206+B216*zeta(1)+B226*zeta(2);

% Solve LMI's for controller in iteration 1 sol 4 is used, whil sol5 is
% used in the following iterations to correct the solution when feasp
% gives a non-negative definite solution. In that case the old solution
% is retained.

if iter<2
[Y2 W2 maxe2 x2]=sol4(A2,B12,B22,C,D2);
[Y3 W3 maxe3 x3]=sol4(A3,B13,B23,C,D2);
[Y4 W4 maxe4 x4]=sol4(A4,B14,B24,C,D2);
[Y5 W5 maxe5 x5]=sol4(A5,B15,B25,C,D2);
[Y6 W6 maxe6 x6]=sol4(A6,B16,B26,C,D2);
else
[Y2 W2 maxe2 x2]=sol5(A2,B12,B22,C,D2,x2);
[Y3 W3 maxe3 x3]=sol5(A3,B13,B23,C,D2,x3);
[Y4 W4 maxe4 x4]=sol5(A4,B14,B24,C,D2,x4);
[Y5 W5 maxe5 x5]=sol5(A5,B15,B25,C,D2,x5);
[Y6 W6 maxe6 x6]=sol5(A6,B16,B26,C,D2,x6);
```

```
end;

me=[maxe2 maxe3 maxe4 maxe5 maxe6];
lam=[lam; me];

%Solve LMI's for optimal zeta
R112=A02*Y2+Y2*A02'+B202*W2+W2'*B202'+(C*Y2+D2*W2)'*(C*Y2+D2*W2);
R113=A03*Y3+Y3*A03'+B203*W3+W3'*B203'+(C*Y3+D2*W3)'*(C*Y3+D2*W3);
R114=A04*Y4+Y4*A04'+B204*W4+W4'*B204'+(C*Y4+D2*W4)'*(C*Y4+D2*W4);
R115=A05*Y5+Y5*A05'+B205*W5+W5'*B205'+(C*Y5+D2*W5)'*(C*Y5+D2*W5);
R116=A06*Y6+Y6*A06'+B206*W6+W6'*B206'+(C*Y6+D2*W6)'*(C*Y6+D2*W6);

lm=[];
lm=add_lmi(lm,[6 2],8);
lm=add_lmi(lm,[6 2],8);
lm=add_lmi(lm,[6 2],8);
lm=add_lmi(lm,[6 2],8);
lm=add_lmi(lm,[6 2],8);

lm=add_lmi(lm,[1],1);
lm=add_lmi(lm,[1],1);

lm=add_var(lm, 1, [1 1]);
lm=add_var(lm, 1, [1 1]);

lm=add_term(lm, [1 1 1 0], R112);
lm=add_term(lm, [1 1 1 1], B212, W2, 's');
lm=add_term(lm, [1 1 1 1], A12, Y2, 's');
lm=add_term(lm, [1 1 1 2], B222, W2, 's');
lm=add_term(lm, [1 1 2 0], B102);
lm=add_term(lm, [1 1 2 1], B112);
lm=add_term(lm, [1 2 2 0], -1);

lm=add_term(lm, [2 1 1 0], R113);
lm=add_term(lm, [2 1 1 1], B213, W3, 's');
lm=add_term(lm, [2 1 1 1], A13, Y3, 's');
lm=add_term(lm, [2 1 1 2], B223, W3, 's');
lm=add_term(lm, [2 1 2 0], B103);
lm=add_term(lm, [2 1 2 1], B113);
lm=add_term(lm, [2 2 2 0], -1);

lm=add_term(lm, [3 1 1 0], R114);
lm=add_term(lm, [3 1 1 1], B214, W4, 's');
lm=add_term(lm, [3 1 1 1], A14, Y4, 's');
lm=add_term(lm, [3 1 1 2], B224, W4, 's');
```

```
lm=add_term(lm, [3 1 2 0], B104);
lm=add_term(lm, [3 1 2 1], B114);
lm=add_term(lm, [3 2 2 0], -1);

lm=add_term(lm, [4 1 1 0], R115);
lm=add_term(lm, [4 1 1 1], B215, W5, 's');
lm=add_term(lm, [4 1 1 1], A15, Y5, 's');
lm=add_term(lm, [4 1 1 2], B225, W5, 's');
lm=add_term(lm, [4 1 2 0], B105);
lm=add_term(lm, [4 1 2 1], B115);
lm=add_term(lm, [4 2 2 0], -1);

lm=add_term(lm, [5 1 1 0], R116);
lm=add_term(lm, [5 1 1 1], B216, W6, 's');
lm=add_term(lm, [5 1 1 1], A16, Y6, 's');
lm=add_term(lm, [5 1 1 2], B226, W6, 's');
lm=add_term(lm, [5 1 2 0], B106);
lm=add_term(lm, [5 1 2 1], B116);
lm=add_term(lm, [5 2 2 0], -1);

lm=add_term(lm, [-6 1 1 1], 1, 1);
lm=add_term(lm, [-7 1 1 2], 1, 1);

[copt,xopt]=linobj(lm,c,[0 0 0 50 1],zeta);

iter
zeta

%Controllers for each flight condition
K2n=W2*inv(Y2);
K3n=W3*inv(Y3);
K4n=W4*inv(Y4);
K5n=W5*inv(Y5);
K6n=W6*inv(Y6);
K2=[K2; K2n];
K3=[K3; K3n];
K4=[K4; K4n];
K5=[K5; K5n];
K6=[K6; K6n];

Cost=c*xopt
cost=[cost; Cost];
zet=[zet; zeta'];
xopt
```

```
zeta=xopt;

if abs(Cost-Costold)/Costold<0.0001
if max(me)<0
break;
end;
end;

% Closed loop system in the final version
Ac2=A2+B22*K2n;
Ac3=A3+B23*K3n;
Ac4=A4+B24*K4n;
Ac5=A5+B25*K5n;
Ac6=A6+B26*K6n;

damp(Ac2)
damp(Ac3)
damp(Ac4)
damp(Ac5)
damp(Ac6)

D=zeros(3,3);

end
```

```
function [Y, W, maxe, x]=sol4(A,B1,B2,C,D)
% Solves the H-inf control problem and returns Y, W matrices
% the maximum eigenvalue and the solution vector in LMI format
lm=[];
lm=add_lmi(lm,[6 6],12);
lm=add_lmi(lm,[6],6);
lm=add_var(lm, 1, [6 1]);
lm=add_var(lm, 2, [3 6]);
lm=add_term(lm,[1 1 1 0], B1*B1');
lm=add_term(lm, [1 1 1 1], A, eye(6), 's');
lm=add_term(lm, [1 1 1 2], B2, eye(6), 's');
lm=add_term(lm, [1 2 1 1], C);
lm=add_term(lm, [1 2 1 2], D);
lm=add_term(lm, [1 2 2 0], -1);
lm=add_term(lm, [-2 1 1 1], eye(6), eye(6));
[tmin,xfeas]=feasp(lm,[200 0 100 1],0);
evalsys=eval_lmi(lm,[xfeas]);
[lhs,rhs]=show_lmi(evalsys,1);
Y=dec2mat(lm,xfeas,1);
W=dec2mat(lm,xfeas,2);
maxe=max(eig(lhs))
x=xfeas;
```

```
function [Y, W, maxe, x]=sol5(A,B1,B2,C,D,x1)
% Solves the H-inf control problem given A, B1, B2, C, D matrices
% and the previous LMI solution. This function checks if all the
% eigenvalues are negative and if they are not returns a solution based
% on the previous iteration.
lm=[];
lm=add_lmi(lm,[6 6],12);
lm=add_lmi(lm,[6],6);
lm=add_var(lm, 1, [6 1]);
lm=add_var(lm, 2, [3 6]);
lm=add_term(lm,[1 1 1 0], B1*B1');
lm=add_term(lm, [1 1 1 1], A, eye(6), 's');
lm=add_term(lm, [1 1 1 2], B2, eye(6), 's');
lm=add_term(lm, [1 2 1 1], C);
lm=add_term(lm, [1 2 1 2], D);
lm=add_term(lm, [1 2 2 0], -1);
lm=add_term(lm, [-2 1 1 1], eye(6), eye(6));

evalsys=eval_lmi(lm,[x1]);
[lhs,rhs]=show_lmi(evalsys,1);
maxe1=max(eig(lhs))

[tmin,xfeas]=feasp(lm,[200 0 100 1],0);
evalsys=eval_lmi(lm,[xfeas]);
[lhs,rhs]=show_lmi(evalsys,1);
maxe=max(eig(lhs))

if maxe<0
 x=xfeas;
else
 maxe=maxe1;
 x=x1;
end;

Y=dec2mat(lm,x,1);
W=dec2mat(lm,x,2);
```

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